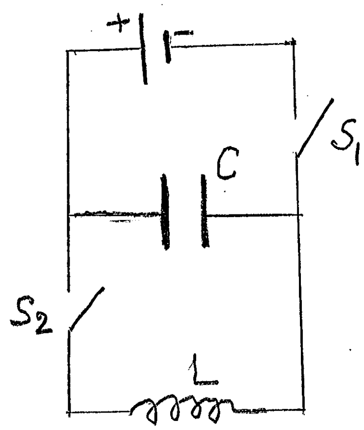
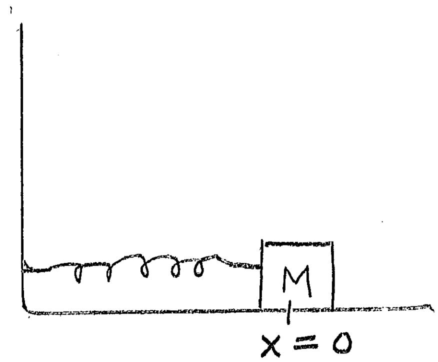
Problems: Week 10

10-1. The pictures show two oscillators.



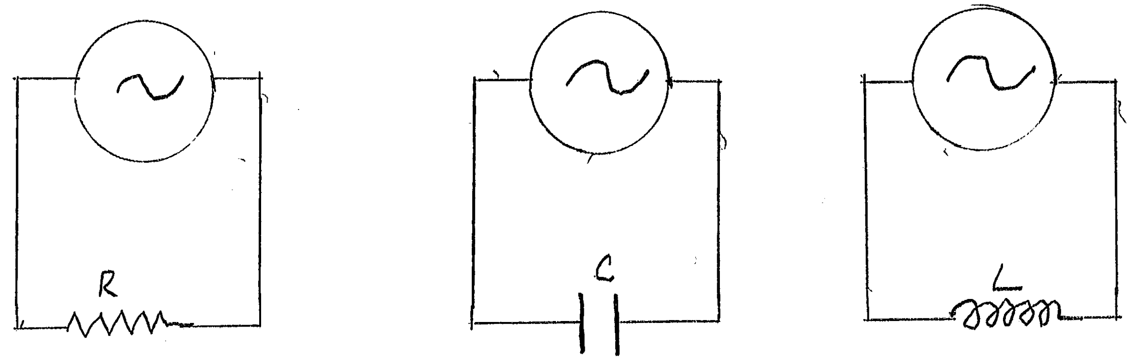
Electrical (El)



Mechanical (Me)

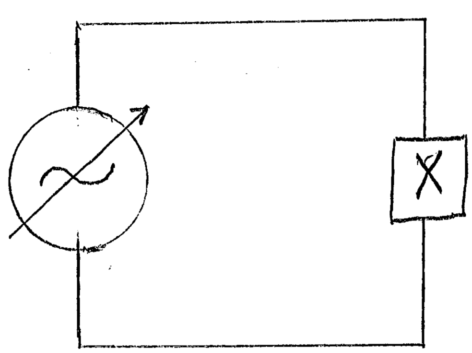
In Me the mass is at rest at $x = 0$ and the spring is relaxed. We pull M to $x = A$ and release it, thereby causing oscillations. In El, first S_1 is closed, capacitor is charged to Q, then S_1 is opened and S_2 is closed, again oscillations ensue. Write down the energy equations for the two systems and explain why the oscillations occur.

10-2. Shown are three circuits.



In each case the output of the ac generator is $\varepsilon = \varepsilon_m \sin \omega t$. We have learned that the currents are $i_R = \frac{\varepsilon_m}{R} \sin \omega t$, $i_C = \varepsilon_m C\omega \cos \omega t$, $i_L = \frac{-\varepsilon_m}{\omega L} \cos \omega t$. Draw appropriate diagrams to show that (i) i_R is in phase with ε, (ii) i_C is $\frac{\Pi}{2}$ ahead of ε and (iii) i_L is $\frac{\Pi}{2}$ behind ε.

10-3. The output of the generator is $\varepsilon = \varepsilon_m \sin \omega t$, ε_m is fixed but the frequency can be set by you. X is an unknown element. Identify X if on doubling the frequency the current (i) increases by a factor of 2, (ii) reduces by a factor of 2, (iii) remains unaltered.

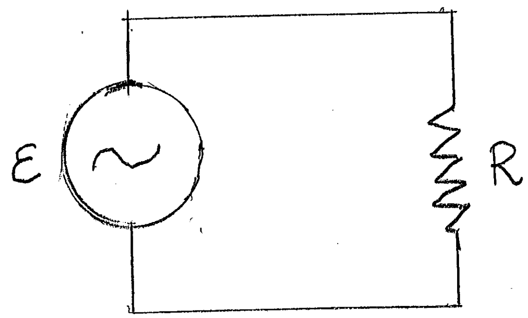


10-4. $\varepsilon = (150 \sin \omega t)$ Volts

$R = 100\Omega$

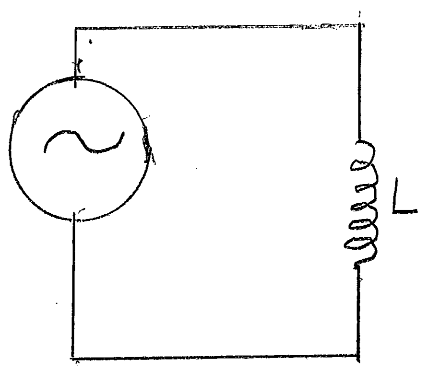
Calculate

1. Root-mean square (r.m.s) voltage
2. Root mean square current
3. Energy absorbed by R per second.



10-5. The ac in your house has an r.m.s. voltage of 110-115V. What is the peak (maximum) voltage?

10-6. If $\varepsilon = 5(\sin \omega t)$ Volts and $f = 600Hz$ and $i_m = 10^{-3}\,amp$ what is L? How much energy is absorbed by L in one cycle? Why?

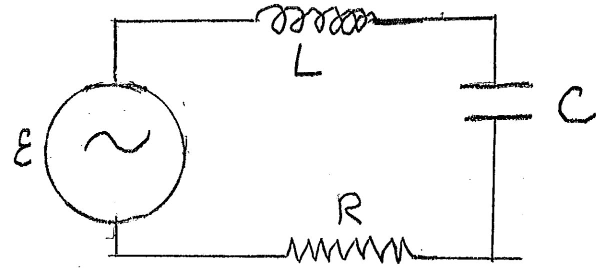


10-7. In the circuit of problem 10-6 replace L by C. If the current remains the same what is C?

10-8. In the circuit shown the current amplitude is

$$i_m = \frac{\varepsilon_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

If you vary ω, $i_m \to 0$ both when $\omega \to 0$ or $\omega \to \infty$. Why?



10-9. In the circuit of 10-8, the current is

$$i = i_m \sin \omega t$$

the potential is

$$v = \varepsilon_m \sin(\omega t + \Phi)$$

where

$$\tan \Phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

and

$$i_m = \frac{\varepsilon_m}{Z} \text{ with } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Show that the average power absorbed is

$$< P >= \frac{\varepsilon_m^{\ 2}}{2R} \cos^2 \Phi$$

Hints: Averages $< \sin^2 \omega t >= \frac{1}{2}$, $< \sin \omega t \cos \omega t >= 0$

10-10. When Maxwell looked at the field equations

$$\Sigma_C \underset{\rightarrow}{B} \cdot \underset{\longrightarrow}{\Delta \ell} = \mu_0 \Sigma I_i \qquad \text{-(1)}$$

and

$$\Sigma_C \underset{\longrightarrow}{E_{NC}} \cdot \underset{\longrightarrow}{\Delta \ell} = -\frac{\Delta \Phi_B}{\Delta t} \qquad \text{-(2)}$$

where $\Phi_B = \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{A}$ flux of $\underset{\rightarrow}{B}$ through area $\underset{\rightarrow}{A}$ and $\underset{\longrightarrow}{E_{NC}}$ is the non-coulomb $\underset{\rightarrow}{E}$ surrounding $\underset{\rightarrow}{A}$; he claimed that Eq (1) was "incomplete". Do you notice the reason for his concern?

10-11. What is the difference between a conduction current and a displacement current?